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# The effect of the fibre critical length on the thermal expansion of composite materials

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The present study shows that both longitudinal and transverse thermal expansions of unidirectional composite materials depend on the length of the fibre. It is suggested that this dependence is through an efficiency factor $k$, generally used for describing the effectiveness of the reinforcement as expressed by mechanical properties such as modulus and strength. $k$ itself is determined by the ratio of the fibre critical length, $l_c$, to the fibre length, $l$, and is expressed by two relationships, one for $l \geqslant l_c$, and another for $l \leqslant l_c$.

The proposed theory is compared with experimental results obtained for samples comprising continuous short fibres. The length of the fibres was varied by varying the size of the test specimens. The length of the fibre is shown to affect significantly the value of the thermal expansion coefficients. An average $l_c$ value calculated from the experimental results is in close agreement with one found by other investigators from tensile strength testing. It is suggested that the experimental procedure may provide a useful tool for assessing the fibre critical length.

## 1. Introduction

Thermal expansion of composites can be predicted by various relationships as a function of constituent material properties. These relationships depend on the type of the composite, i.e. particulate or fibrous. Several approaches to the theoretical study of thermal expansion of particulate composites exist, and have been reviewed recently [1]. Expressions for coefficients of thermal expansion of unidirectional fibrous composites have been derived by Schapery [2] and examined by other investigators [3, 4]. These expressions do not take into account the fibre length, and it is the purpose of the present study to investigate this issue.

## 2. The effect of the critical length

Thermal expansion coefficients of fibrous composites are given by Equations 1 and 2 which have been derived for longitudinal and transverse expansions, respectively [2, 3].

$$\alpha_c{}^L = \frac{E_m\alpha_m V_m + E_f\alpha_f V_f}{E_m V_m + E_f V_f} \tag{1}$$

$$\alpha_c{}^T = (1 + \nu_m)\alpha_m V_m + (1 + \nu_f)\alpha_f V_f - \alpha_c{}^L(\nu_f V_f + \nu_m V_m) \tag{2}$$

where $\alpha$ is the coefficient of thermal expansion, $E$ is Young's modulus, $\nu$ is Poisson ratio and $V$ is the volume fraction, L and T denote longitudinal and transverse directions, and the subscripts, f, m and c, denote fibre, matrix and composite properties, respectively. Equations 1 and 2 are based on the consideration that thermal expansion, being a dilatation strain, depends on the axial restraint of the fibres. In the longitudinal direction, low expansions are obtained due to the fibres constraining the matrix from expanding. This constraint results in squeezing the matrix out in the transverse direction. The extent of the axial restraint depends on the shear strength of the fibre–matrix bond and on their interfacial area, since these factors control the fibre–matrix and matrix–fibre stress transfer mechanism. Thus, the axial restraint is related to the critical length* of the fibre, the value of which is also determined by those factors.

*The fibre critical length is defined by the expression $l_c = d\sigma_f/2\tau_i$ where $d$ is the fibre diameter, $\sigma_f$ is the fibre strength and $\tau_i$ is the fibre matrix shear strength.

Recent studies [5-8] describe the relations between mechanical properties such as modulus and strength of fibrous composites and the fibre critical length. For composites reinforced by short discontinuous fibres, an efficiency factor, $k$, is introduced into these relations. The efficiency factor describes the effectiveness of the reinforcement as follows:

$$\sigma_c = \sigma_m V_m + k\sigma_f V_f \quad (3)$$

$$E_c = E_m V_m + kE_f V_f \quad (4)$$

$$\epsilon_c = \frac{E_m \epsilon_m V_m + kE_f \epsilon_f V_f}{E_m V_m + kE_f V_f} \quad (5)$$

Where $\sigma$ and $\epsilon$ express stress and strain, respectively. In the case of uniaxially aligned short discontinuous fibres, the value of $k$ is given by the following equations [6]:

$$k = \frac{l}{2l_c} \ (l \leqslant l_c\,,\ 0 \leqslant k \leqslant 0.5) \quad (6)$$

$$k = 1 - \frac{l_c}{2l} (l \geqslant l_c\,,\ 0.5 \leqslant k \leqslant 1) \quad (7)$$

where $l$ is the length and $l_c$ is the critical length of the fibres.

The thermal expansion of composite materials is affected by the shear transfer process at the interface in a similar way to that experienced by a mechanical strain. Hence, by analogy to Equations 1 and 5, the following equation for longitudinal expansion may be derived:

$$\alpha'_c{}^L = \frac{E_m \alpha_m V_m + kE_f \alpha_f V_f}{E_m V_m + kE_f V_f}\,. \quad (8)$$

Similarly, the transverse expansion may be represented by Equation 9:

$$\alpha'_c{}^T = (1 + \nu_m)\alpha_m V_m + (1 + \nu_f)\,\alpha_f V_f - \alpha'_c{}^L(\nu_f V_f + \nu_m V_m) \quad (9)$$

where $\alpha'_c{}^L$ and $\alpha'_c{}^T$ are the longitudinal and transverse coefficients, respectively, of the thermal expansion of a uniaxially aligned short discontinuous fibre composite.

Fig. 1 presents plots calculated from Equations 8 and 9 of $\alpha'_c{}^T$ and $\alpha'_c{}^L$ as functions of $k$. The data used for compiling these plots is given in Section 4. It is seen that the value of $\alpha'_c{}^L$

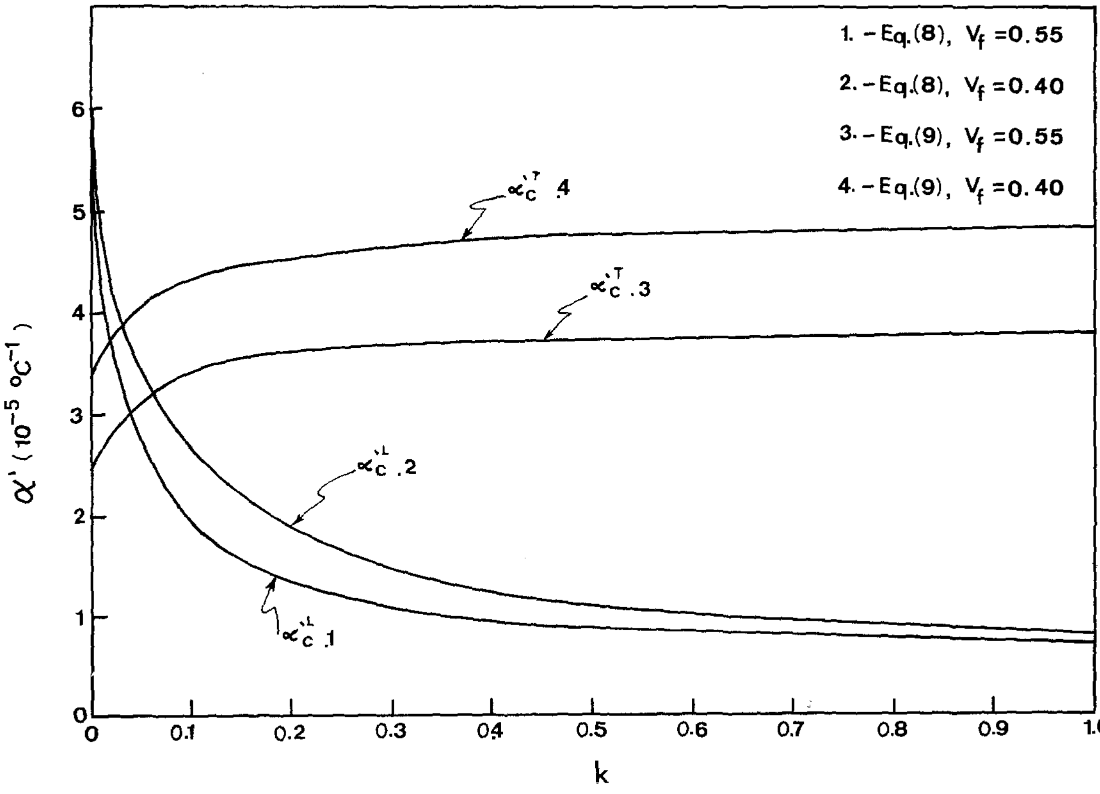


*Figure 1* Plots of Equations 8 and 9 for $V_f = 0.40$ and 0.55, showing $\alpha'_c{}^L$ and $\alpha'_c{}^T$ as a function of $k$.

increases as $k$ decreases. At the point $k = 0$ (zero efficiency) $\alpha'_c{}^L = \alpha_m$. The value of $\alpha'_c{}^T$, however, decreases with $k$, in agreement with the anticipation that a smaller restraint of the fibres results in a lower transverse expansion.

## 3. Experimental procedure

### 3.1. Materials

Epoxy resin*-glass fibre† composites of $V_f = 0.53$ and $V_m = 0.45$ were manufactured as follows. "Pre-pregs" were prepared by winding and impregnating the fibres on a winding machine drum. From the dry "pre-preg" a number of 13 cm × 13 cm sheets were cut. These sheets were laid-up in a three plate mould, and pressed at 180°C under 2.76 MN m$^{-2}$ (400 psi) to yield a 13 cm × 13 cm unidirectional composite plate 1.2 mm thick. $V_f$ was controlled by the number of sheets introduced into the mould.

### 3.2. Sample preparation and measurement of expansivity

Measurements of thermal expansion coefficients were carried out by a stainless steel apparatus similar to that described elsewhere [9]. The apparatus was calibrated by means of an electrolytic copper specimen of known expansivity ($\alpha = 1.692 \times 10^{-5}$ °C$^{-1}$).

Thermal expansion measurements were made over the range room temperature to 110°C. The temperature was raised slowly during a period of 4 h at an average rate of 0.38°C min$^{-1}$. Fig. 2 represents some of the expansion versus temperature plots. As seen, the curves obtained are linear except at the first point (room temperature). This point was the only one to be measured at a steady state. A value of $\alpha$ was calculated from the slope of each line. The slope was worked out by a linear regression analysis which excluded the first point.

The following example describes the procedure of sample preparation and testing. A 5 cm × 5 cm specimen comprised of continuous fibres was cut from the original composite plate. The thermal expansivity was measured in both longitudinal and transverse directions. The sample was then transversely cut to produce identical rectangular plates of a shorter fibre length. All of these plates were axially held in series by glass clamps for a longitudinal expansivity measurement. A close contact between the plate edges was ensured. Some of these plates were used

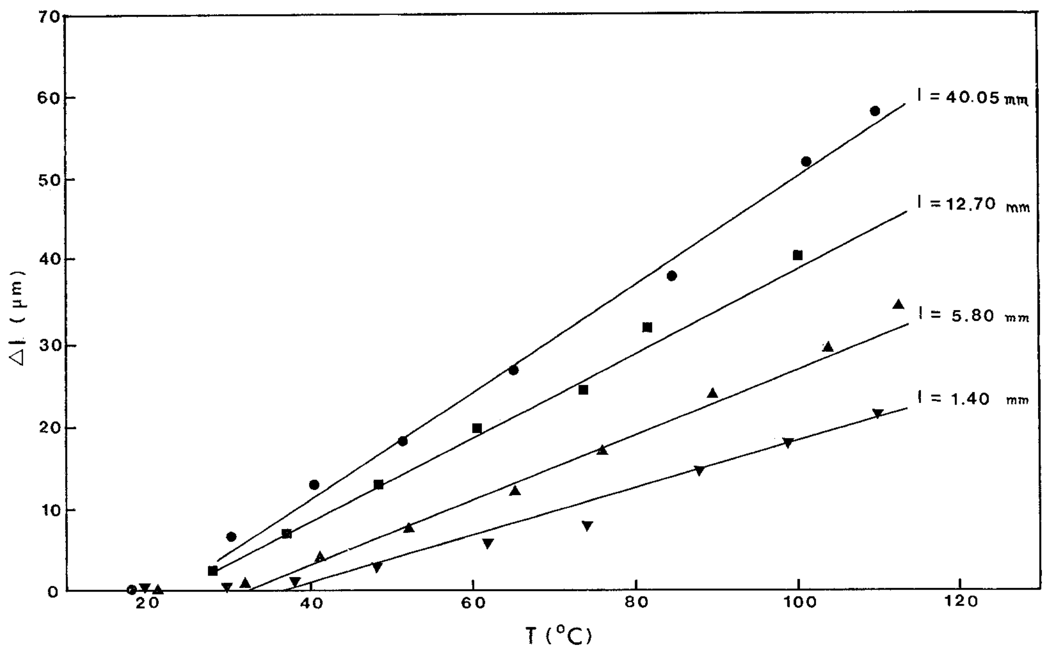


*Figure 2* Plots of transverse thermal expansion versus temperature. The slope decreases as the fibre length is decreased.

*Ciba-Geigy Ltd, Araldite MY750 cured by the hardener HT972.
†Vetrotex, EC14-300, K937.

separately for measuring the transverse expansivity. The rectangular plates were then sliced in half and the thermal expansivities were measured similarly. This process was repeated a few times.

## 4. Results and discussion

In this study the length of continuous fibres was varied by changing the size of the test specimen, rather than using test specimens of short discontinuous fibres. This enabled an examination of the effect of the fibre length on the expansivity with specimens made out of the same plate, and having identical fibre volume fractions. A previous attempt to use composites of short discontinuous fibres yielded highly scattered results, since it was difficult to produce samples of an identical fibre volume fraction.

The experimental results were compared with Equations 1, 2, 8 and 9 using the following parameters: $E_f = 70$ GN m$^{-2}$, $E_m = 3.0$ GN m$^{-2}$, $\nu_f = 0.2$, $\nu_m = 0.35$, $\alpha_f = 0.5 \times 10^{-5}$ °C$^{-1}$, $\alpha_m = 6.0 \times 10^{-5}$ °C$^{-1}$, $V_f = 0.53$, $V_m = 0.45$.

Figure 3 presents the results of the coefficients of transverse thermal expansion as a function of the fibre length. The square and the circle symbols indicate two separate sets. This figure also compares these results with values predicted by Equations 2 and 9, the latter being drawn for various values of $l_c$ ($l_c$ = 5, 10, 15, 20 mm). It is seen that the value of $\alpha'_c{}^T$ for the lower values of $l$ is smaller than the value set by Equation 2. However, as $l$ is increased, the value set by Equation 2 is slightly exceeded. The $\alpha_c{}'^T$ results of samples of shorter $l$ lengths are significantly lower than those of samples of longer $l$ lengths. The trend of the experimental results generally agrees with that predicted by Equation 9. The fibre critical length of the examined composites shows best agreement with the predicted curves for $l_c$ values between 10 and 20 mm.

The results obtained for the coefficients of longitudinal thermal expansion are presented as a function of the fibre length in Fig. 4. These results are also compared with values set by Equation 1 and 8, the latter being plotted for a few values of $l_c$ ($l_c$ = 5, 10, 15, 20 mm). It is seen that $\alpha'_c{}^L$ values increase as $l$ decreases. The

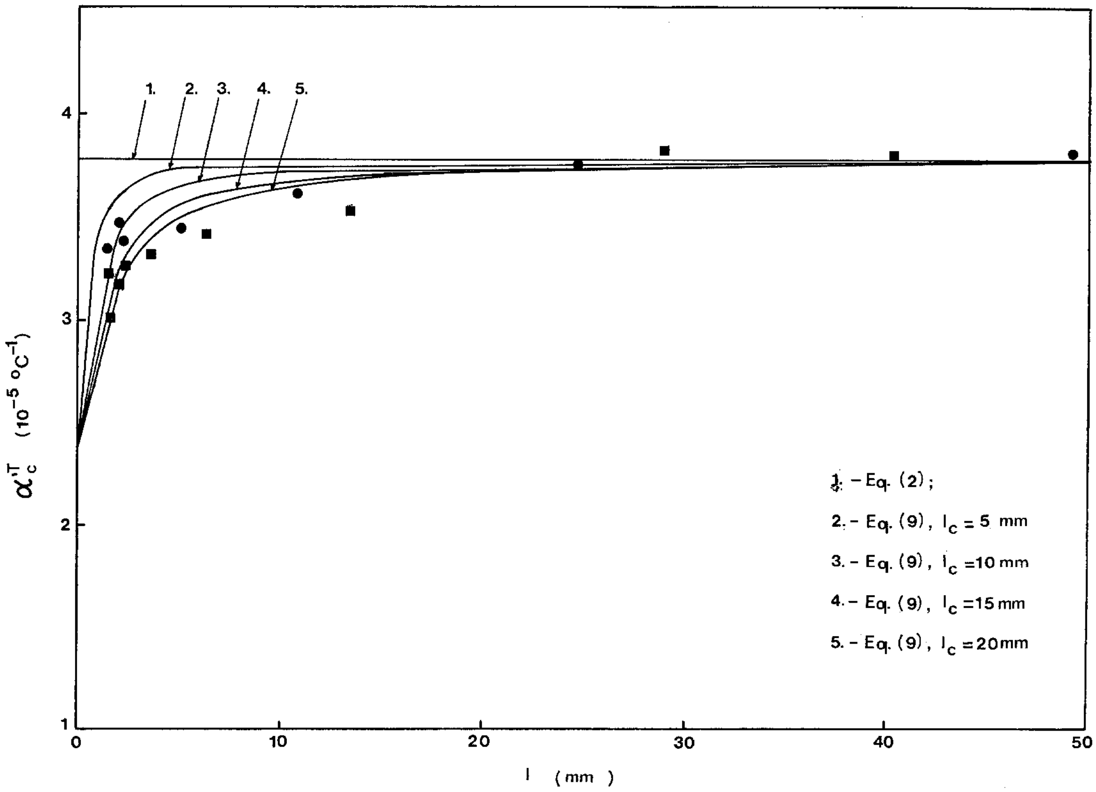


*Figure 3* $\alpha'_c{}^T$ values as a function of the fibre length. A comparison between experimental values and the theoretical expressions drawn for various values of $l_c$.

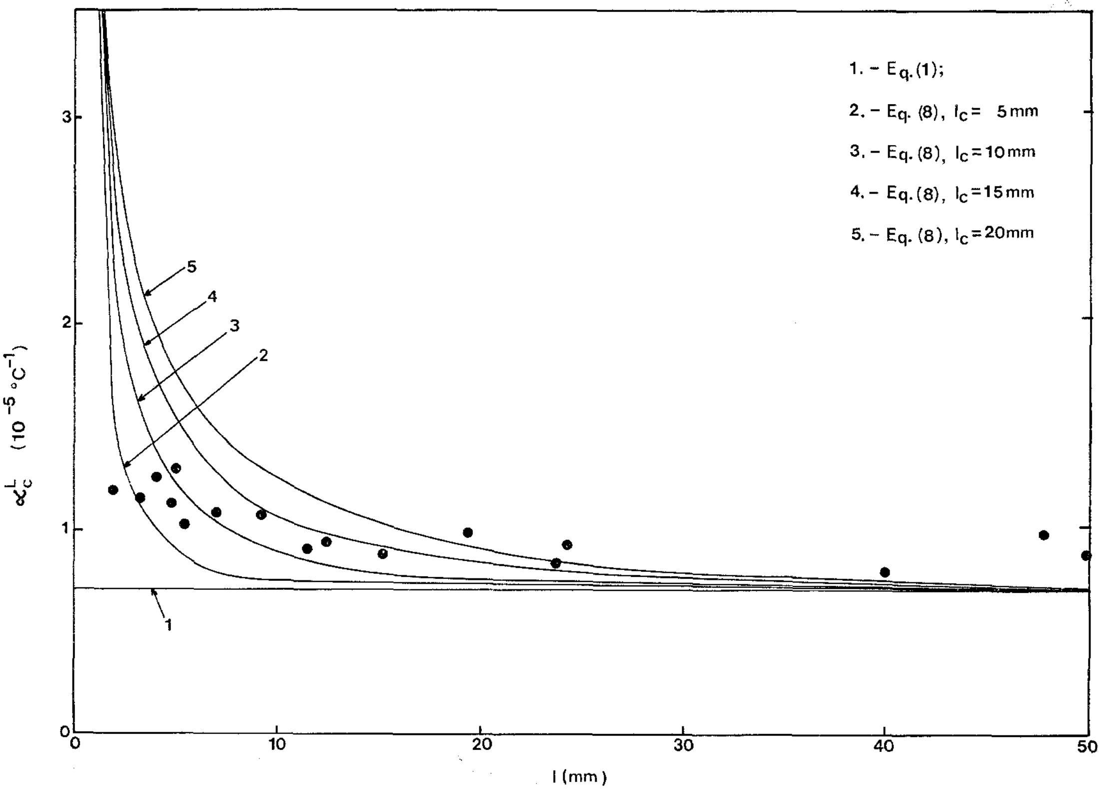


*Figure 4* $\alpha'_c{}^L$ as a function of the fibre length. A comparison between experimental values and the theoretical expressions drawn for various values of $l_c$.

best agreement between experimental $\alpha_c{}'^L$ values and those of Equation 8 is observed over the range $5 < l < 15$ mm. However, experimental data do not exhibit any tendency to increase rapidly at short lengths. This may result from the difficulty in the longitudinal testing of specimens thinner than 2 mm.

The average experimental $l_c$ value for samples of fibre length in the range 1 to 25 mm was calculated as follows. The $l_c$ values were calculated from Equation 8 using experimental $\alpha'_c{}^L$ values. Similarly, they were calculated from Equation 9 using experimental $\alpha'_c{}^T$ values. The average $l_c$ result for the longitudinal samples was 14.0 mm with a scatter of approximately 50%. The average $l_c$ result for the transverse samples was 24.2 mm with a scatter of approximately 50%. This gives a final average $l_c$ value of 19.1 mm.

A value of $l_c$ in the range 10 to 20 mm is plausible for epoxy resin-glass fibre composites of the type examined in the present study, where no special care was taken to improve the fibre–matrix interfacial bond. This value is in close agreement with the value of 12.7 mm obtained by Hancock and Cuthbertson [5] for similar materials. In their work the ultimate tensile strength of uniaxially aligned short discontinuous fibre composites was examined as a function of the fibre length.

## 5. Conclusions

There is experimental indication supporting the theory suggested for the effect of the fibre critical length on the thermal expansion of fibre reinforced composites. The range of $l_c$ derived from the experimental results agrees with a value found by a mechanical testing procedure [5]. This suggests that the shear transfer process at the fibre–matrix interface affects thermal strains in a similar way to mechanical strains.

The experimental procedure described in this study may provide a useful tool for assessing the fibre critical length in composite systems.

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